Entropy Kinetics during Self-Organization Process in Open System

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ABSTRACT: The process of self-organization in an open steady nonequilibrium system is considered. The behavior of a local dissipative zone of system is investigated by using a model of a bistable element and the mathematical apparatus of the Markovian stochastic process theory. Formulas, which describe kinetics of the entropy flow and its rate, are obtained with respect to random process of influences exerted upon a system. It was revealed that the open system responds to a sudden change of conditions by steep growth of the entropy flow up to a maximum value at the critical point (being the stochastic analog of a bifurcation point). The model makes it possible to describe the whole life cycle for a system, including passage across a sequence of the bifurcation points ("jumps" of development) and evolutionary stages of development at each hierarchical level on a unified basis.

Nomenclature

D	: Variance
dS	: Variation of entropy
d_eS	: Entropy flow through a system (entropy exchange between system and environment)
d_iS	: Entropy production inside a system
dH/dt	: Entropy flow rate
F(X), F(Y)	: Probability distribution function for instantaneous values of random processes X(t) and Y(t) respectively
H	: Information entropy
k	: Bolzmann constant
LDZ	: Local dissipative zone of an open system
$P_0(t)$, $P_1(t)$	: Probability of down and up states of bistable element respectively
P_0, P_1	: Final (independent of time) probability of down and up states of bistable element respectively
R	: Universal gas constant
S	: Thermodynamic entropy
t	: Time
t_0	: Initial moment of time
t_b	: Moment of time, corresponding to a bifurcation point
U, U*	: Initial and subsequent sets of external and/or internal being conditions of a system
X	: Determining parameter of a system
$\overline{X}$	: Mean (expected) value of the determining parameter X
X(t)	: Random process of the determining parameter changing in the LDZ of a system
Y(t)	: Random process of external influences exerted upon a system

Greek Symbols

$\alpha=\nu/\mu$	: Regime parameter
$\beta=\nu+\mu$	: Sum of transition rates between the two states
ε_1, ε_2, ε_3	: First, second, and third principal strains average over a LDZ respectively

ε_i, σ_i	: Intensity of strain and stress respectively
$\Phi[Z]$	: Cumulative normal (Gaussian) distribution function of the random variable Z
$\mu(t)$, $\nu(t)$	: Transition rate from up state to down state and vice versa
σ_1, σ_2, σ_3	: First, second, and third principal normal stresses average over a LDZ respectively
σ_m	: Mean value of stress
σ_S	: Yield strength

Subscripts

0	: Down state of bistable element
1	: Up state of bistable element
th	: Threshold level
max	: Maximum value
ST	: Value, corresponding to steady (stable) state

Superscript

*	: Value, corresponding to subsequent sets of external and/or internal being conditions of a system

1. Introduction

The study of the reasons, principles and mechanisms of the self-organization in an open system of various natures (technical, biological, socio-economic, ecological, etc.) is a subject of the new interdisciplinary scientific direction, named synergetics. The term "synergetics," developed by H.Haken, [1,2] emphasizes the importance of concerted action in the process of self-organization. Synergetics usually deals with the open steady nonequilibrium active systems - dissipative systems. The spectrum of such systems is quite broad.

Recall that a system may be open or closed (isolated), depending on whether or not it exchanges matter and energy with its environment. [3,4] If this exchange of energy and matter is structured (or codified), we can designate it as a transfer of information (entropy). In this broad sense, information exchange is not necessarily limited to "intelligent" systems. [5] Presence of an energy flux from an external source to a system and the dissipation of energy on external environment are the preconditions of activity in any system. Because of this, the evolution of open active systems does not necessarily lead towards the equilibrium (the most chaotic) state. On the contrary, open systems may be involved in processes of self-organization, which result in more complicated and more advanced structures.

An open system that we are going to deal with is macroscopic. This means that the system (that is macroscopic level) consists of a number of subsystems (mesoscopic level), that in turn consists of a number of elements (microscopic level), which can be atoms and molecules in physics and chemistry, cells and microorganisms in biology, groups of living organisms in sociology, etc.

As is known, [3,6-8] for open systems the variation of the entropy dS for an interval of time dt can be decomposed into a sum of two components

$$dS = d_eS + d_iS \quad (1)$$

These have quite different physical meanings. d_eS is the entropy flow, which depends on the processes of matter and energy exchange between system and environment. d_iS is the entropy production, caused by irreversible processes inside the system. According to the second law of thermodynamics, d_iS can only be positive or vanish in the absence of irreversible processes. Component d_eS may be invariably zero, positive, or negative in terms of the energy received (or given up) by the system. If conditions $d_eS<0$ and $|d_eS|>d_iS$ are observed, the certain stages of temporal evolution in the open system can occur at general downturn of entropy $dS<0$.

It is necessary to note that such a situation is possible only far from equilibrium, as in an equilibrium state the member d_iS always prevails. It means that a system is so far from its equilibrium that the linear laws no longer apply; nonlinear terms become important. Self-organization is the "supercritical" phenomenon. [9] Nevertheless far from equilibrium, the system may still evolve to some steady state. In far from equilibrium conditions, various types of self-organization processes may occur.

According to the traditional interpretation of entropy, as a measure of disordering (uncertainty) of a system, it should be considered that, if the disorder decreases at the expense of entropy return (outflow) in the course of evolution, the system evolves to more complicated and more advanced structures. [8,9] Thus self-organization, i.e., spontaneous formation of high-ordering structures is possible by virtue of an energy flux which passes through the system.

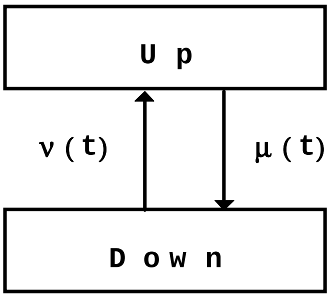
2. Basic Ideas and Definitions

Let an open system be in a steady (stable) nonequilibrium state. Further, let us assume that in this state the positive entropy production is compensated for a negative entropy flow

$$d_iS = -d_eS \qquad (2)$$

i.e., the system gives back so much entropy as is produced inside the system. The condition (2) establishes a condition of current balance between the entropy flow through the system and its production inside the system in a steady (stable) nonequilibrium state. Current balance is understood as stationary (not time-dependent) nonequilibrium state of an open system, stable in relation to small deviations. The system in this state is actually in dynamic balance with its environment. The processes of inflow and outflow in a system at current balance are always balanced; however, as $d_iS>0$, the power processes proceeding in the system are always dissipative, i.e., are accompanied by dissipation of energy. The dissipation of energy is an essential main attribute of current balance. One of main attributes of steady nonequilibrium systems is their ability to use a surplus of free energy for fulfillment of the expedient functions, directed at maintaining an initial nonequilibrium state of a system. As appear, the dissipative system is efficient (live) so long as it is capable of releasing itself from the entropy which it produces during the dissipation of energy. An exchange with the environment can be maintained by the system itself only when its internal state is far from equilibrium; otherwise, the processes would die down. The entropy production inside system in certain sense character-

Fig. 1. Graph of transition in the bistable element modelling the LDZ behavior



izes exhaustion of the system lifetime. Formulas, describing kinetics of the entropy flow and its rate, will be deduced below. According to Eq. (2), these terms can be extended to the internal entropy production inside the system in a steady (stable) nonequilibrium state. [10]

All real processes are irreversible and unbalanced in some degree. Local gradients of temperature, chemical potential, pressure (special case of which is the stress and strain concentration) can exist only in a nonequilibrium system, and, if the system is closed, they relax in the course of time (evolution towards equilibrium). Let X be an important local parameter, determining longevity, lifetime, load-carrying capacity or any other property of vital significance for an open system in a local zone (hereafter we shall call X as determining parameter). Further, let us label a zone of an increased gradient (or a local concentration) of the determining parameter as the local dissipative zone (LDZ). Such a local zone limits the lifetime of a system. As a rule, these zones correspond to places of the most probable failures during its lifetime. In the course of intensive operating, the number of LDZ can grow. The growth dynamics of the LDZ number is an important property of the behavior of dissipative system; however, in this paper this question is not considered. Here we investigate the evolution of a system based on processes, occurring only in one LDZ.

At least two auto-regulating mechanisms of energy dissipation, corresponding to two hierarchical levels (two stable states), can act inside a system. We shall consider the behavior of a LDZ by using the model of a bistable element. Bistable element (Fig. 1) has two stable states (down and up), in each of which it can exist for a rather long time. Let us denote the mean value of determining parameter of a system in the down state as X_0 and in up state as X_1. External influences or internal changes in a system can result in transition of the bistable element from one state to another. To cause the transition, an intensity of influence should exceed some threshold level. It was assumed that the transition from down state to up state is caused by such an external influence exerted upon a system, at which determining parameter X (average over a volume of LDZ) exceeds mean (expected) value of the threshold level X_{th}.

In each particular case, the physical content of determining parameter, its threshold level and criterion of transition from one state to another are defined by the type and nature of the system examined and depend on the statement of a problem. In part 6.1 we shall consider interpretation of this concept for elasto-plastic deformation of metal in the local zone of a load-carrying structure.

3. Mathematical Formulation

For studying a behavior of the LDZ of an open system, mathematical apparatus of the theory of

Markovian stochastic processes was used. Recall that a random process with discrete states and continuous time is referred to as a Markovian, if its state at a particular time depends on its state at the immediate preceding time but not on any of its states at earlier times. [11-13] In other words, a Markovian process is characterized by absence of an after-effect. However, the Markovian property of random process does not mean complete independence of the future from the past. In a general case in Markovian process, "the future depends on the past only through the present." [13]

The differential Kolmogorov equations for the probabilities $P_0(t)$ and $P_1(t)$ of respectively the down and up states of the bistable element were written in the following form [13]

$$\frac{dP_0(t)}{dt} = -\nu(t)P_0(t) + \mu(t)P_1(t); \tag{3}$$

$$\frac{dP_1(t)}{dt} = \nu(t)P_0(t) - \mu(t)P_1(t) \tag{4}$$

In the case of a homogeneous Markovian process where the transition rates do not depend on time (that is $\nu(t) = \nu$; $\mu(t) = \mu$), and under the condition that at the initial moment of time t_0, the LDZ is in the down state (that is $P_0(t_0)=1$; $P_1(t_0)=0$), the solution of differential equations (3) and (4) is shown in the following form [13]

$$P_0(t) = \left(1 + \alpha e^{-\beta t}\right) / (1 + \alpha) \tag{5}$$

$$P_1(t) = \alpha\left(1 - e^{-\beta t}\right) / (1 + \alpha) \tag{6}$$

where $\alpha=\nu/\mu$; $\beta=\nu+\mu$.

Hereafter we shall call α regime parameter since it defines the mode of the LDZ being,

The Markovian process is ergodic, because at $t\to\infty$ there are final probabilities of the down and up states corresponding to the stationary stage. They are equal [13]

$$P_0 = \lim_{t\to\infty} P_0(t) = 1/(1+\alpha);$$
$$P_1 = \lim_{t\to\infty} P_1(t) = \alpha/(1+\alpha). \tag{7}$$

This means that after completing the transient process, at the stationary stage the LDZ of a system will change its state transferring from down state to up state and vice versa, but their probabilities no longer depend on time. They can be interpreted as the mean relative time of the LDZ stay in the individual states. From Eqs. (5) to (7) it follows that the process in the LDZ approaches the stationary stage exponentially.

Depending on the regime parameter α three characteristic "mode of being" [14] the LDZ are considered: light at $\alpha<1$, $\nu<\mu$, $P_0>P_1$, i.e., the LDZ is for a long period of time in the down state; symmetric at $\alpha=1$, $\nu=\mu$, $P_0=P_1=0.5$; heavy at $\alpha>1$, $\nu>\mu$, $P_1>P_0$, i.e., the LDZ is for a long period of time in the up state.

Let us show that the system evolving in accordance with differential equations (3) and (4) is dissipative. [15] For this purpose, let's analyze a vector $\overline{X} = \{P_0 P_1\}$ and vectorial function $\overline{V}(\overline{X}) = \{V_0 V_1\}$ with components V_0 and V_1, equal for system of differential equations (3), (4) to the following functions

$$V_0 = -\nu P_0 + \mu P_1; V_1 = \nu P_0 - \mu P_1 \tag{8}$$

If a condition [15]

$$div\overline{V}(\overline{X}) < 0 \quad (9)$$

is satisfied, the system is dissipative. We obtained the result

$$div\overline{V}(\overline{X}) = \sum_{j=0}^{1} \frac{\partial V_j}{\partial X_j} = \frac{\partial V_0}{\partial X_0} + \frac{\partial V_1}{\partial X_1} = -(\nu + \mu) < 0 \quad (10)$$

which confirms the fact that the researched system is dissipative.

4. General Results

The behavior of the statistical properties of the examined bistable element in the course of evolution is in detail analyzed in the articles. [10,16] Let us list only some principal properties.

Time dependence of mean (expected) value $\overline{X}(t)$ of the determining parameter was calculated under the obtained formula

$$\overline{X}(t) = \left[X_0 + \alpha X_1 - \alpha(X_1 - X_0)e^{-\beta t}\right]/(1+\alpha) \quad (11)$$

At transient stage, it has the tendency to grow or decrease depending on the initial conditions, as shown in Fig. 2 by solid line. Curve 1 occurs when the initial state is the down one and curve 2 -- when the initial state is the up. The dashed line corresponds to the steady level of the mean value of determining parameter $\overline{X}_{ST}$. One can observe that both lines 1 and 2 approached the steady level at stationary stage.

The steady level is independent of initial conditions and obtained in the form

$$\overline{X}_{ST} = \left(X_0 + \alpha X_1\right) / (1 + \alpha) \quad (12)$$

The second of the principal statistical properties we studied is variance of the determining parameter X. The time course of a variance $D_X(t)$ was described by term

$$D_X(t) = (X_1 - X_0)^2\left[P_0(t) - P_0^2(t)\right] \quad (13)$$

The graph of time courses of the variance $D_x(t)$ for heavy and light modes of the system being is shown in Fig. 3. Since in the initial moment of time $P_0(t_0)=1$, we have $D_X(t_0)=0$. At the stationary stage (at $t \to \infty$) we have

$$D_{ST} = \left(X_1 - X_0\right)^2 \frac{\alpha}{(1+\alpha)^2} \quad (14)$$

Let us consider the critical point of the variance function (13). First and second derivative tests for critical point of this function can be written in the form

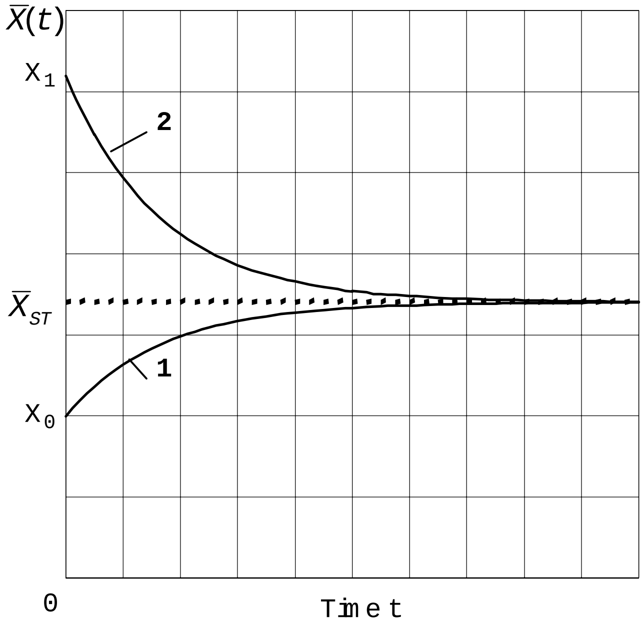


Fig. 2. The time courses of the mean value $\overline{X}(t)$ of determining parameter (solid lines) from down (1) and up (2) initial states. Steady level $\overline{X}_{ST}$ is denoted by a dashed line.

$$\frac{dD_X(t)}{dt} = 0; \; \frac{d^2D_X(t)}{dt^2} < 0 \tag{15}$$

Eq. (15) is satisfied for heavy mode of being at α>1 in a critical point, corresponding to a moment

$$t_b = -\frac{1}{\nu+\mu}\ln\left(\frac{\alpha-1}{2\alpha}\right). \tag{16}$$

In this critical point variance function (13) has a maximum, equal to

$$D_{\max} = \frac{1}{4}\left(X_1 - X_0\right)^2 \tag{17}$$

This critical point corresponds to a stochastic analog of a bifurcation point of a system. [8]

The light mode of the LDZ being when α<1 (See line 2 in Fig. 3) is characterized by absence of the critical point and by stabilization of the variance on the level D_{ST} in the course of evolution towards a stationary stable state.

Let us assume that the random process Y(t) of dynamic influences with probability distribution function F(Y), is exerted upon a system. This process produces a similar random process X(t) of changing the determining parameter X in the LDZ. Further, let us assume the probability distribution function for instantaneous values of the random process is F(X). A dependence of regime parameter α, which defines the mode of the LDZ being, on the statistical properties of this process is obtained in the following form

$$F(X_{th}) = P_0 = 1/(1+\alpha) \qquad 1 - F(X_{th}) = P_1 = \alpha/(1+\alpha) \tag{18}$$

In example (See, part 6.1) we shall obtain functions connecting regime parameter α and statistical

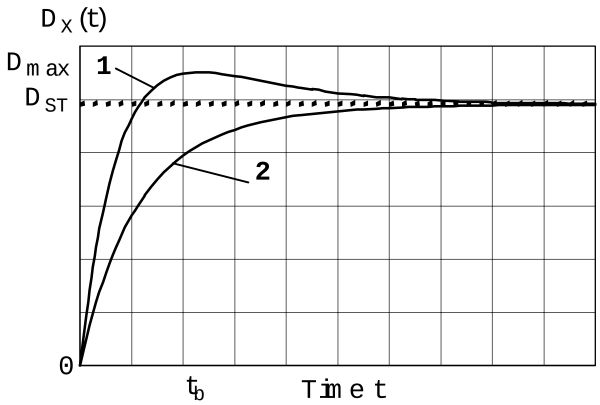


Fig. 3. The time courses of the variance $D_X(t)$ for heavy (1) and light (2) modes of being (solid lines). Steady level D_{ST} is denoted by a dashed line. The coordinates of the critical point are labeled as D_{max} and t_b.

properties (mean value and variance) of normal (Gaussian) random process exerted upon a system.

One of the most important variables of any system is entropy. The main point of interest of the foregoing analysis is the possibility of computing and forecasting the time course of entropy. Note that various interpretations of entropy are internally linked quite closely. [4] The physical entropy of a system coincides with the thermodynamic entropy S. The information entropy H is connected to them by a ratio [17]

$$S = k \ln 2 \times H \tag{19}$$

The Eq. (19) can be rewritten per unit of amount of a substance (1 mole) in the form

$$S = R \ln 2 \times H \tag{20}$$

The information entropy, being the measure of uncertainty, [18] equal to the amount of information according to Shannon, required for removing this uncertainty, will be examined.

For the bistable element modelling the LDZ, the information entropy function H is determined from the equation [18]

$$H = -\sum_{j=0}^{1} p_j(t) \log_2 p_j(t) \tag{21}$$

The minimum value of entropy H=0 corresponds to the degeneration of a stochastic system into a rigid determinate system. The maximum H_{max}=1 in a closed system corresponds to its transition in an equilibrium state, that equivalently of its wreck. For open self-organizing systems, this value corresponds to a moment of bifurcation, when there is the destruction of pattern (microstructure) exhausting its dissipative abilities, and resulting in an emergence of new pattern at other hierarchical levels.

Taking into account decomposition of entropy into a sum of two components in Eq. (1) and condition (2) of current balance, we shall consider kinetics of entropy flow in the LDZ of a system.

Eqs. (21), (5) and (6) were used to obtain the analytical dependence of the entropy flow on time. After some algebra we obtained following formula

$$H(t) = -\frac{\alpha}{1+\alpha}\frac{1}{\ln 2}\left\{\frac{1+\alpha e^{-\beta t}}{\alpha}\ln\left[\frac{1}{1+\alpha}\left(1+\alpha e^{-\beta t}\right)\right] + \left(1-e^{-\beta t}\right)\ln\left[\frac{\alpha}{1+\alpha}\left(1+e^{-\beta t}\right)\right]\right\} \quad (22)$$

The time courses of the entropy flow H(t) is shown in Fig. 4. Analysis of function (22) shows that in heavy mode of the LDZ being (at $\alpha>1$) at the time t_b, determining by term

$$t_b = -\frac{1}{\nu+\mu}\ln\left(\frac{\alpha-1}{2\alpha}\right) \quad (23)$$

the maximum entropy flow is reached, $H(t_b)=1$. It can be shown that this moment corresponds to the condition of equal probabilities (maximum uncertainty) of keeping the LDZ in the down and up states

$$P_0(t_b) = P_1(t_b) = 0.5 \quad (24)$$

Note that the moment of time t_b, determined by Eqs. (23), (24) corresponds to the critical point of maximum variation D_X, determined by formula (16). That means in the case of instability, some fluctuations get amplified up to a macroscopic scale during macroscopic times. In this way, fluctuations -- environmental perturbations or eigenfluctuations -- may drive the system into a completely new state and thus become the driving force of system development ("order through fluctuation"). Instabilities at the same time can break symmetry, that is, bifurcations occur: the system may choose among two states, though determined by causality. The "selection" of the future path of development is unpredictable now. [19] However, at least we can predict the moment of maximum uncertainty and endeavor to take precautions against an unfavorable path of development.

After passing the system through the critical point which is a stochastic analog of the bifurcation point, [8] the entropy flow decreases and, leaving the transient stage, stabilizes on the steady (stable) level

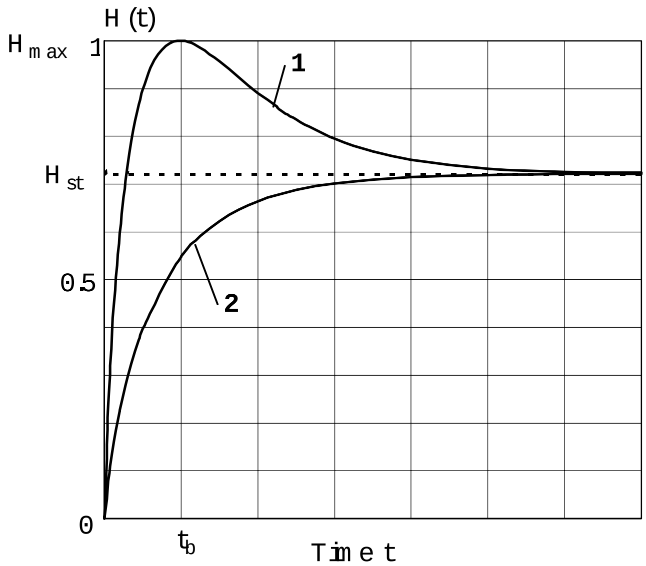


Fig. 4. The time courses of the entropy flow H(t) for heavy (1) and light (2) modes of being (solid lines). Steady level H_{st} is denoted by a dashed line. The coordinates of the critical point are labeled as H_{max} and t_b

$$H_{st} = \ln\left[(1+\alpha)\Big/\alpha^{\frac{\alpha}{\alpha+1}}\right]/\ln 2 \qquad (25)$$

According to the traditional interpretation of entropy, it means that because of entropy outflow in the course of evolution, the disordering decreases. A system is structuring in response to the heavy being mode by self-organization of more complicated and more advanced structures (patterns). This is an attribute of the system's adaptation to the random process characterized by the regime parameter $\alpha>1$.

The light being mode of the LDZ when $\alpha<1$ (See Figure 4) is characterized by absence of the stochastic analog of the bifurcation point and by stabilization of the entropy flow on the level H_{ST} during the period of exit from the transient to stationary stage.

From positions of the synergetics, special interest is attracted by kinetics of the entropy flow rate in the course of the system's evolution. The entropy flow rate is defined as the derivation of the entropy flow function (22). It was obtained (10,16,20) as a Zainetdinov formula in the following form

$$H' = \frac{dH}{dt} = -\frac{ve^{-\beta t}}{\ln 2}\ln\left[\frac{\alpha\left(1-e^{-\beta t}\right)}{1+\alpha e^{-\beta t}}\right] \qquad (26)$$

Plot of the entropy flow rate with time is shown in Fig. 5. Analyzing the function (26) allows us to conclude that the response of a system to the heavy being mode ($\alpha>1$) by a rapid increase of the entropy flow takes place simultaneously with reduction of the entropy flow rate to zero at the moment of time t_b (see Fig. 5). Hereafter the rate of entropy flow becomes negative, passes through a minimum, and aspires to zero, when the transient process approaches the steady (stable) stage. It corresponds to the I.Prigogine [3] theorem of minimum entropy production during the stationary process in system.

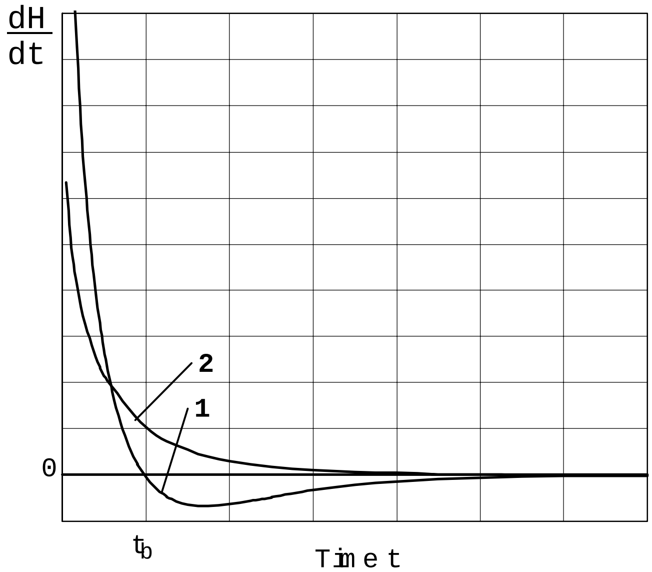


Fig. 5. The time courses of the entropy flow rate dH/dt for heavy (1) and light (2) modes of being

5. Influence of the variation of conditions

Consider now behavior of the LDZ with the variation δU of external and/or internal conditions of a system. The main point of interest of the foregoing analysis is the possibility of computing and forecasting a system response (namely the time course of entropy) to a change of the being conditions. Denote an initial set of conditions by U, a subsequent set by U*=U+δU, and new values of the quantities, corresponding to set of conditions U* letters with an asterisk as well. The change of conditions may lead to both the modification of the regime parameter α* and the probabilities of states P_0* and P_1*. For fulfillment of the analysis we used a system of differential Kolmogorov equations, similar to Equations (3) and (4). Timing of time t* started again at the moment of variation of the conditions U*. The initial conditions were formulated by using the final probabilities (4) in the following manner

$$P_0^*(t_0^* = 0) = P_0 = \frac{\mu}{\nu + \mu} = \frac{1}{1+\alpha} \tag{27}$$

$$P_1^*(t_0^* = 0) = P_1 = \frac{\nu}{\nu + \mu} = \frac{\alpha}{1+\alpha} \tag{28}$$

Solution of the Kolmogorov differential equations for the probabilities P_0*(t*) and P_1*(t*) of respectively the down and up states of the bistable element of the LDZ under new conditions U* with respect to initial conditions (27), (28) were obtained in the following form

$$P_0^*(t^*) = \frac{1}{1+\alpha^*} + \left(\frac{\alpha^*}{1+\alpha^*} - \frac{\alpha}{1+\alpha}\right)e^{-\beta^* t^*} \tag{29}$$

$$P_1^*(t^*) = \frac{\alpha^*}{1+\alpha^*} + \left(\frac{\alpha}{1+\alpha} - \frac{\alpha^*}{1+\alpha^*}\right)e^{-\beta^* t^*} \tag{30}$$

where β*=ν*+μ*.

In the asymptotic limit, as t→∞, the final probabilities of the down and up states corresponding to the stationary stage under new conditions U* were obtained as follows

$$P_0^* = \frac{\mu^*}{\nu^* + \mu^*} = \frac{1}{1+\alpha^*} \quad \text{and} \quad P_1^* = \frac{\nu^*}{\nu^* + \mu^*} = \frac{\alpha^*}{1+\alpha^*} \tag{31}$$

One can note that these final probabilities do not depend on initial conditions (27) and (28).

From the positions of synergetics, an important role is played by analysis of the functions of entropy flow H*(t*) and its rate dH*/dt* as a response of an open system to a sudden change of the external and/or internal conditions. Using equations (21), (29) and (30), mathematical expression for the time course of the entropy flow H*(t*) under new conditions U* was obtained in following form

$$H^*(t^*) = -\frac{1}{\ln 2(1+\alpha)(1+\alpha^*)}\Big\{\left[(1+\alpha)+(\alpha^*-\alpha)e^{-\beta^* t^*}\right]\times$$
$$\times \ln\frac{(1+\alpha)+(\alpha^*-\alpha)e^{-\beta^* t^*}}{(1+\alpha)(1+\alpha^*)} + \left[\alpha^*(1+\alpha)+(\alpha-\alpha^*)e^{-\beta^* t^*}\right]\times \quad (32)$$
$$\times\ln\frac{\alpha^*(1+\alpha)+(\alpha-\alpha^*)e^{-\beta^* t^*}}{(1+\alpha)(1+\alpha^*)}\Big\}$$

A function of the entropy flow rate after a variation of conditions of the system existence was received in the following form

$$\frac{dH^*(t^*)}{dt^*} = \frac{\nu\mu^* - \nu^*\mu}{\beta\ln 2}e^{-\beta^* t^*}\ln\left[\frac{\nu^*\beta+(\nu\mu^*-\nu\mu)e^{-\beta^* t^*}}{\mu^*\beta+(\nu^*\mu-\nu\mu^*)e^{-\beta^* t^*}}\right] \quad (33)$$

The initial rate of the entropy flow is equal to

$$\left.\frac{dH^*(t^*)}{dt^*}\right|_{t^*=0} = \frac{\nu\mu^* - \nu^*\mu}{\beta\ln 2}\ln\alpha \quad (34)$$

It is evident that the obtained functions (32) to (34) depend on the combination of the previous U and new U* being conditions.

Fig. 6 shows the graphs of the functions of entropy flow H*(t*) and its rate dH*/dt* for the case when being conditions become more heavy ($\alpha<1$, $\alpha^*>1$). On the first time interval ($0<t<1.4$ s) when $\alpha<1$, the LDZ exists in an easy being mode, so the entropy flow is stabilized at a level H_{ST}, corresponding to this mode. At the time t=1.4 s being conditions become heavier ($\alpha^*>1$). An open system responds to a sudden change of the external and/or internal conditions by a rapid increase of the entropy flow from the stationary level H_{ST}, obtained under previous being conditions, to the maximum value $H^*(t_b^*)=1$. At the time t_b^*, the rate of entropy flow sharply falls up to zero.

The mathematical expression for moment of time t_b^* was obtained in the following form

$$t_b^* = -\frac{1}{\nu^*+\mu^*}\ln\frac{(\alpha^*-1)(\alpha+1)}{2(\alpha^*-\alpha)} \quad (35)$$

One can note, that it depends on a combination of the previous and new being conditions U, U*.

In a time t_b^* after transition in a heavier being mode ($\alpha^*>1$) the LDZ passes across a bifurcation point. This point is associated with destruction of the pattern (dissipative structure) of the first hierarchical level, exhausting its dissipative possibilities, and emergence of new appropriate pattern, corresponding to the changed being conditions. Leaving on the second level of hierarchy after the jump of development, the LDZ enters an evolutionary stage of development. There is the rather slow stabilization of the entropy flow during this stage at the expense of saturation by the information up to a level $I=1-H_{ST}^*$, which corresponds to a new mode of system existence. In other words, the LDZ adapts to new being conditions by perfection of structure (that is self-organization). In this case, the entropy flow rate (See, Fig. 6, b) gets negative value, passes through a minimum and, remaining negative, aspires to zero, when the transient process

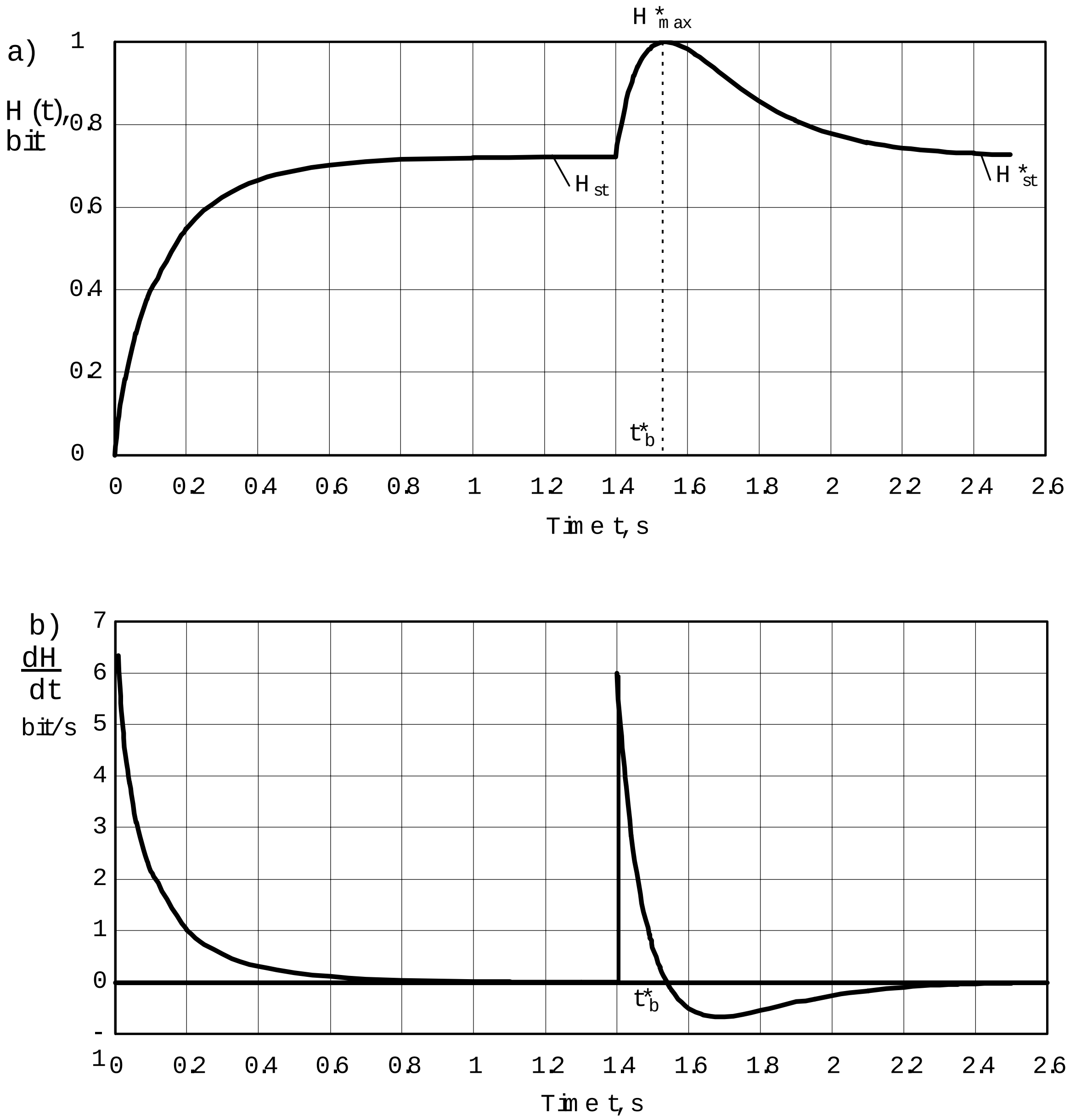


Fig. 6. Response of a system to a change of the being conditions.

The time courses of the entropy flow H(t) (a) and its rate dH/dt (b) for the case when mode of being becomes heavier ($\alpha<1$, $\alpha^*>1$). The coordinates of the critical point are labeled as H^*_{max} and t^*_b.

reaches the stationary stage with a new steady level of the entropy flow H_{ST}^*, adequate to conditions U*. It corresponds to the Prigogine [(3)] theorem of the minimum entropy production during a stationary process in a system.

In passage through the critical point at time t_b^* the variance D^*_X of the systems determining parameter X is maximum. The system is characterized at this stage by the highest degree of disordering, with the random fluctuations manifested on the macroscopic level. After passage through the critical point, the variance D^*_X is stabilized on a new stationary level corresponding to the being conditions U*.

Unfortunately, there is no place for more detailed examination of behavior in the course of time of the statistic properties of the systems determining parameter X after changing the being conditions. The mathematical terms, describing the time courses of these statistic properties (such as mean value $\overline{X}^*(t^*)$ and variance $D^*_X(t^*)$) may be found in articles. [10,16] Let us note only, that there are possible both the cases of monotonous increase or decrease of the variance D^*_X and the cases of emerging the characteristic peak on a curve of variance D^*_X. The characteristic peak is similar to that shown in Fig. 3 and corresponds to a critical (bifurcation) point. Passing through such a characteristic peak and stabilizing the variance D^*_X on a new steady level D^*_{ST} confirm completion of the period of running in ("burn-in") of a system and its adaptation to new changed conditions. When testing the system, it is possible to measure and record the time history of the mean value and variance of determining parameter in the vicinity of some LDZs. Analysis of processes, occurring in the LDZ, on a basis of considered criterion makes it possible to speed up the time of reception data about adaptation of system to new changed conditions.

An open system of any nature comes back in a steady stable state due to inflow of the information from the outside and redistribution of the information entropy between hierarchical levels of the system. [21]

After passage through the bifurcation point, the entropy flow decreases in accordance with an information accumulation; that means appropriate increase of an organization level during a system development. At each hierarchical level of a system's evolution at the end of self-organization process, when the "architecture" of the system was basically defined and becomes saturated by the information, the entropy curve is gradually straightened, displaying the transition of a system to the evolutionary stage of development. A growth of the organization degree for any system has limit, area of saturation, determined by limited opportunities of an information accumulation in given structure at a given hierarchical level.

Such a picture of temporal evolution of an open system with respect to variation of the being conditions agrees with the synergetic qualitative approach to processes of self-organization in nonequilibrium dissipative systems. [1-9,22] Obtained analytical dependencies reveal quantitative ratio, reflecting kinetics of the entropy flow and its rate, which shows evolution of open systems in the course of its life cycle under complex variation of being conditions. Revealed laws, in our opinion, are applicable for open nonequilibrium systems of various natures: technical, economic, biological, ecological, social, etc. The discussed model and obtained dependencies make it possible on a unified basis to describe the whole life cycle of a system, including passage across a sequence of bifurcation points ("jumps" of development) and evolutionary stages of development at each hierarchical level. Transition to a new level of development goes from disorder to order, through the phenomenon of instability in the bifurcation points, where a system has a number of options to diverge in several directions.

In the following, we shall give a few examples in order to elucidate the received results.

6. Examples

6.1. Elasto-plastic deformation of metal in local zone of structure

Let us consider the self-organization process during elasto-plastic deformation of metal in local zones of load-carrying structures. (16,23-27) From the thermodynamical and synergetic point of view, a material undergoing plastic deformation is an open system brought far from equilibrium conditions. (28,29) In this case, the LDZ is represented by the zone of stress (strain) concentration, and the dissipative system is the load-carrying structure as a whole. Let us accept as determining parameter X, limiting the lifetime of the structure, stress intensity

$$\sigma_i = 1/\sqrt{2}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad (36)$$

or strain intensity

$$\varepsilon_i = (\sqrt{2}/3)\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2} \qquad (37)$$

determined from the distortion energy (Huber - Von Mises - Hencky) theory. (30) It was assumed, that transition from down state, corresponding to elastic behavior of metal in the LDZ, in up (the plastic yielding state) is caused by such an external influence exerted upon system under which determining parameter exceeds a threshold level, equal to the expected value of the yield strength σ_S or the strain of the yield strength ε_S

$$\sigma_i \geq \sigma_S \quad \text{or} \quad \varepsilon_i \geq \varepsilon_S \qquad (38)$$

In the three-dimensional coordinate system σ_1, σ_2 and σ_3, the plasticity surface, corresponding to the Huber - Von Mises - Hencky condition, (30) represents a circular cylinder with the same angle γ in relation to the individual axes, as is shown in Fig. 7. If the trajectory of the determining parameter $\sigma_i(t)$ is situated inside this cylinder, then the LDZ is in down state, and if it is on the plasticity surface, then the LDZ transfers to up state.

The main point of interest of this example is to show how the random loading process exerted upon the load-carrying structure is linked with the structure response (namely the time course of entropy) to a change of the loading conditions. Let us accept the hypothesis that instantaneous values of the loading process have a normal (Gaussian) distribution with the following statistical properties: mean value σ_m and variance D_σ of the stresses. Using Eqs. (18), the relations that link the regime parameter α with these statistical properties and also with the threshold level (mean value of yield stress σ_S) were obtained in form

$$\alpha = \left[\Phi\left(\frac{\sigma_S - \sigma_m}{D_\sigma}\right)\right]^{-1} - 1 \qquad \text{at } \sigma_m<\sigma_S \qquad (39)$$

$$\alpha = \left[1 - \Phi\left(\frac{\sigma_m - \sigma_S}{D_\sigma}\right)\right]^{-1} - 1 \qquad \text{at } \sigma_m>\sigma_S \qquad (40)$$

Fig. 7. Graphical representation of the transition criterion between the elastic and plastic states of metal in the LDZ

where $\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} \exp\left(\frac{-x^2}{2}\right) dx; \; Z = \frac{\sigma_S - \sigma_m}{D_\sigma}$ (41)

The dependence of α on σ_m and D_σ at the threshold level σ_S=310 MPa for low-carbon steel is computed from Eqs. (39) to (41) and plotted in Fig. 8. We used this approach for a load-carrying welded structure, namely the pivoting section of a gondola car body. Examination of the time history of stress change in elements of the gondola car, including directly the welded joints, carried out using the automated system for amplitude-spectral analysis, shows that the probability distribution of the instantaneous values of the random process is normal.

It may be seen that the loading regime of the LDZ of the load-carrying structure is affected mostly by σ_m. At $\sigma_m<\sigma_S$, the light regime ($\alpha<1$) is established; at $\sigma_m>\sigma_S$, the heavy regime ($\alpha>1$) is established. The variance D_σ of the process, regardless of its value, cannot have any strong effect on the being regime.

In the course of elasto-plastic deformation in metals, a number of dislocation patterns (microstructures) are formed. [31] One after the other ball, cellular, persistent slip band, quasi-amorphous microstructures arise and are destroyed, consistently replacing each other on a background of existing grains' boundaries. Further increase of load results in formation of the crack origins in a quasi-amorphous zone and growth of their density. The spontaneous emergence of a quasi-amorphous microstructure corresponds to achievement of a maximum disorder in this local zone, at which point the thermodynamic entropy is maximum and equal to enthalpy of melting. All these transitions are supervised by achievement of a maximum level of the entropy flow S_{max}. [31] Under action of an energy flux, pumped up by the stochastic loading process in the LDZ, the deformation ability of metal at the lowest hierarchical structural level is exhausted. Getting through a critical point of bifurcation, metal passes to a higher level of the pattern hierarchy to microstructure, having the higher dissipative characteristics. [31]

a)

b)

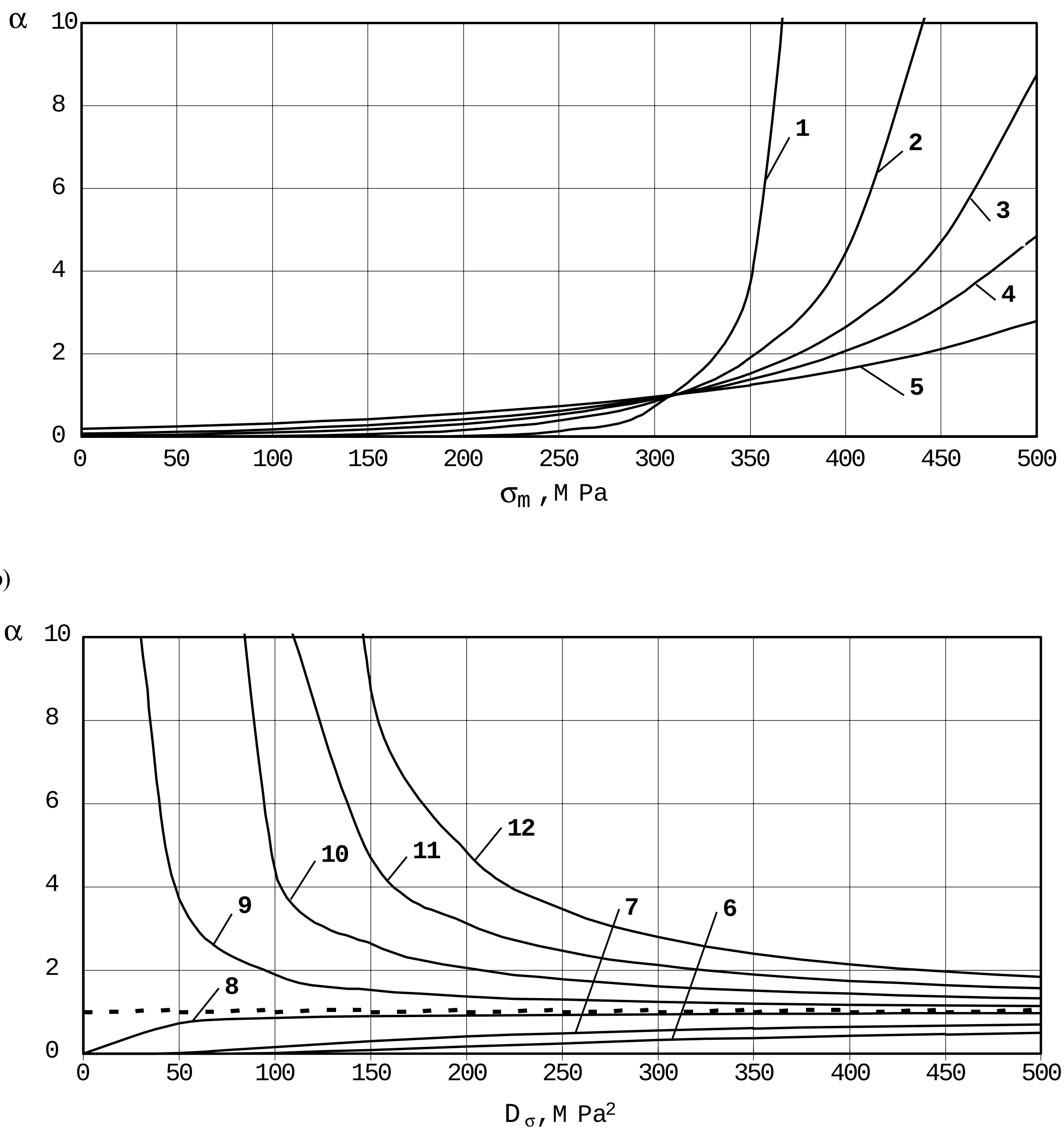


Fig. 8. Change of the regime parameter α as function of the statistical properties (mean value σ_m (a) and variance D_σ (b)) of Gaussian random process of dynamic stresses: 1 -- 5) D_σ is equal to 50, 100, 150, 200, 300 MPa^2 respectively; 6 -- 12) σ_m is equal to 100, 200, 300, 350, 400, 450, 500 MPa respectively. Line α=1, corresponding to threshold level σ_m=σ_S=310 MPa is denoted by dashed line.

This transition between patterns on the microscopic level is reflected by the curves of the entropy flow and its rate, obtained before. Returning to Fig.6, note that at the first time stage (0<t<1.4s) the regime parameter α was determined (by using plots of Fig. 8) to be equal to 0.25. It corresponds to the vertical (gross) dynamic load of the gondola car. The value α*=5 corresponds to the total effect of the vertical and longitudinal inertial (in collision of cars) loads. That mode of the LDZ being matches to the transition of metal in plastic state. In Fig. 6, one can see that a load-carrying structure responds to the heavy conditions by a rapid increase of the entropy flow from the stationary level H_{ST}, obtained under vertical load, to the maximum H^*_{max} at the time t_b^*. This critical point corresponds to the destruction of pattern exhausting its dissipative abilities, and the emergence of new microstructure at another hierarchical level. The system is structuring responding to the heavy being mode by self-organization of more advanced patterns.

6.2. Nicolis & Prigogine's Model

In the book, [8] is considered a process of transition between multiple stationary states caused by formation of embryo in the model system. The plots, showing the time courses of the mean value and variance of fluctuations for this model, are indicated in Fig. 9, which was adopted from. [8] They correspond to analytical dependencies, Eqs. (11), (13) of mean value $\overline{X}(t)$ and variance $D_X(t)$ of determining parameter X with time (See Fig. 2 and 3).

Unfortunately, there was no more place and time for elaborating many special examples and applications. We hope that they will be presented soon in some original publications.

CONCLUSIONS

1. The process of self-organization in an open steady nonequilibrium system is examined. The behavior of a local dissipative zone of system is investigated by using a model of a bistable element and mathematical apparatus of the Markovian stochastic process theory. Formulas, which describe kinetics of the entropy flow and its rate, are obtained with respect to stochastic process of random influences exerted upon a system.

It was revealed that the various kinds of open systems (technical, biological, socio-economic, ecological, etc.) respond to a sudden strong change of external or/and internal being conditions by the same way: by steep growth of the entropy flow up to a maximum value at the critical point. According to the traditional interpretation of entropy, it means that the disordering and chaos in the system increase at this stage of its evolution. The critical point (being the stochastic analog of a bifurcation point) associates with the self-organization process, that is the destruction of pattern (dissipative structure) of the previous hierarchical level, exhausting its possibilities, and resulting in the emergence of new more complicated and more advanced structures (patterns), corresponding to the changed being conditions. In other words, the system can adapt to new being conditions by perfection of its structure.

The model makes it possible to describe the whole life cycle for the system, including passage across a sequence of the bifurcation points ("jumps" of development) and evolutionary stages of development at each hierarchical level on a unified basis.

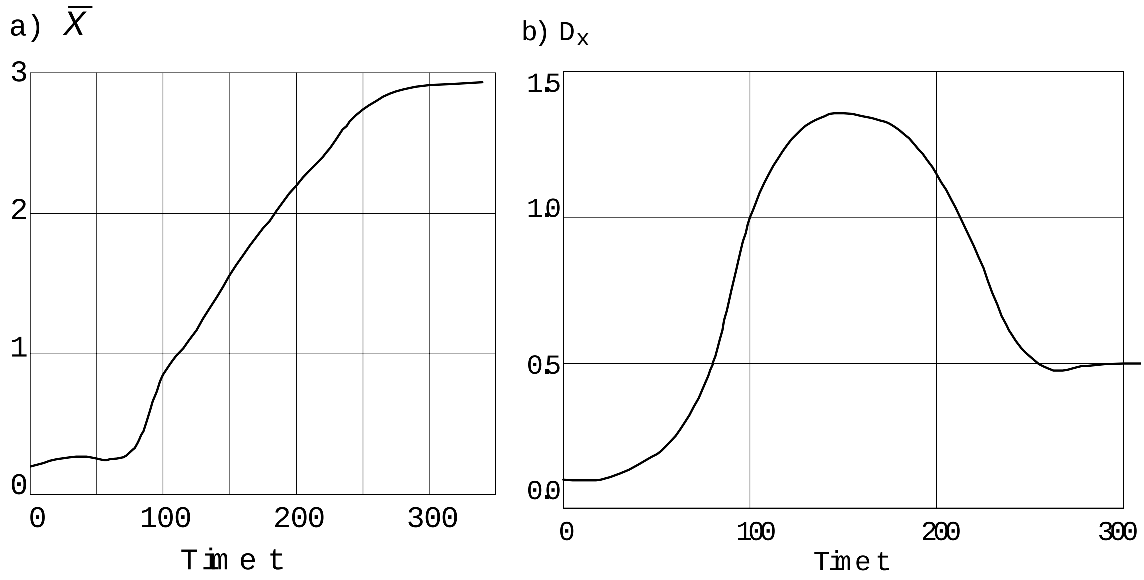


Fig. 9. Evolution of the mean value $\overline{X}$ (a) and variance D_X (b) of fluctuations for the Nicolis-Prigogine's model. Adapted from [7].

Open system of any nature comes back in a steady stable state due to inflow of the information from the outside or/and redistribution of the information entropy between the hierarchical levels of system.

2. Obtained mathematical expressions for the time course of the entropy flow make it possible to predict the moment of the critical point (bifurcation point) coming. That is the moment of maximum uncertainty, instability, and chaos in the system when small fluctuations (environmental perturbations or eigenfluctuations) become amplified up to a macroscopic scale. It may drive the open system into a completely new state and thus become the driving force of system development. At the present time we cannot forecast the "selection" of the future path of development. However, at least we can predict the moment of maximum uncertainty and endeavor to take precautions against unfavorable path of development.

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