J 적분 수치계산법에 의한 응력확대계수 결정

Numerical J integral Method To Determine Two Dimensional Stress Intensity Factors in Cracked Sheets

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Key Words : crack, stress intensity factor, traction singular quarter point element, J integral

ABSTRACT : The traction singular quarter point boundary element having $r^{-1/2}$ and $r^{+1/2}$ behavior of the stress and the displacement near the crack tip is known as an effective means to determine accurately two-dimensional stress intensity factor in linear elastic fracture problems. However, it should be pointed out that the use of small sized traction singular quarter point elements near the crack tip, in particular, under coarse mesh arrangements would cause serious deterioration of stress intensity factor. As an alternative to resolve such difficulty, the J integral method is investigated to find the possibility to reduce errors in evaluating the stress intensity factor when used with small sized traction singular quarter point elements near the crack tip. A J_1 integral for the central crack problem of finite width in plane stress is derived for the practical numerical implementation based on the quadratic interpolation function as well as J_1 integrals for the single and the double edge crack problems. Accurate and less mesh-sensitive numerical results are illustrated for the coarse mesh arrangements with 9 boundary elements.

요약 : 응력확대계수는 J적분에 의해서도 계산할 수 있음이 잘 알려져 있다. 이 방법의 장점은 이론적 정식화에서 알 수 있듯이 균열 선단을 포함하는 영역의 정보가 배제됨에도 불구하고 응력확대계수를 간접적으로 계산할 수 있다는 점이다. 따라서 균열 선단 주위의 모델링 상세 여부에 영향을 덜 받을 수도 있다는 관점에서 연구의 관심을 끈다. 본 연구에서는 균열특이요소를 채용한 경계요소법을 기반으로 균열특이요소에서 직접적으로 얻어지는 응력확대계수를 계산하고 아울러 비교를 위해서 J 적분법에 의해 간접적으로 응력확대계수를 계산 비교하였다. 특이균열요소의 길이(L) 대비 균열 크기의 반(a) 즉 (L/a)를 파라메터로 하여 0.05에서 0.9까

지의 수치 연구를 수행하였다. 연구 결과 균열 크기에 비해 대단히 작은 균열특이요소를 사용한 직접적인 계산법에서 큰 오차를 보여주는 (L/a)≦0.1인 영역에서도 J 적분법이 상대적으로 괄목할만한 적은 오차를 보여주었다. 0.1≦(L/a)≦0.9 영역에서도 J 적분법은 바람직스러운 정밀도와 더불어 파라메터 (L/a)의 값에 별 영향없이 균일한 오차를 보여주었다.

1. Introduction

The accurate prediction of the stress intensity factor (SIF) in cracked sheets has been the most common concern pertaining to linear elastic fracture mechanics. Analytical or approximate techniques have been investigated to determine stress intensity factors in cracked sheets. A few good solutions for certain idealized crack problems are available in closed form. SIF's can be directly determined in terms of the applied stress level, the crack size and the geometric factor from these analytical solutions. When used with such analytical means, however, it is nearly impossible to handle the problems of arbitrary shaped cracked bodies subjected to various loading conditions. An alternative powerful approach may be to use numerical methods such as the FEM (finite element method) or the BEM (boundary element method). Thus, SIF's can be directly extracted using the computed stress and the deformation fields around the crack tip. In related to the former techniques, as demonstrated by Henshell and Shaw[4], and Barsoum[5], crack tip elements were devised to incorporate the stress singularity of $r^{-1/2}$ near the crack tip and were used to evaluate SIF's by the direct displacement regression method.

Continuing research efforts have been also made toward the use of BEM. It is well-known that even under highly coarse element arrangements for variety of crack problems, the BEM yields more accurate SIF's. We cite three important publications. First, Blandford et al.[6] presented an accurate BEM technique using the traction singular quarter point element to model the behavior of the stress and the displacement in the vicinity of the crack tip. The SIF was directly determined from the relationship among the nodal displacements on the crack surface adjacent to the crack tip. For a certain parametric ratio of the crack tip element length over half of the crack length (L/a) ranging from 0.05 to 0.9, results showed good accuracy within a narrow range around 0.1 only when used with the traction singular quadratic element for the double edge crack problem. Second, Martinez and Dominguez[7] developed a more improved technique to calculate SIF's directly from the nodal values of the tractions in the vicinity of the crack. The use of traction singular quarter point elements having $r^{-1/2}$ and $r^{+1/2}$ behavior of the stress and the displacement near the crack tip, respectively, was proven relatively more accurate than other direct methods of such as displacement regression methods. Although such a special modeling technique may be valid only in a small vicinity of a crack tip in its concept, their computed results revealed satisfactory accuracy over the almost range of the parameter (L/a) except the range less than 0.1 for the centercrack and the double edge crack problem. In contrasting, it seems paradoxical that the use of small sized elements near the crack tip within the range of the parameter less than 0.1 still remains prohibited under coarse mesh arrangements. Unlike prior studies, Jia et al.[9] employed, instead of traction singular quarter point elements, alternative special shape functions to model the displacement and the tractions near the crack tip. They reported that results under the same coarse mesh arrangements were less accurate than those of Martinez and Dominguez.[7]

On the other hand, it is noted that SIF's can be calculated indirectly via path independent integrals. The motivation for using such integrals may be due to the fact that the direct evaluation of near tip field quantities can be avoided, yet the value of the integrals is related to the SIF at the crack tip. A substantial formulation and analysis associated with this subject appears to have been conducted by Eischen[8]. Finely meshed finite element scheme employing crack tip modeling techniques [4,5] were used to compute the fields. The problem of the single edge crack in a finite width strip of homogeneous materials was investigated for the parameter of the crack size over the width of the strip (a/W) ranging from 0.0 to 0.8. It was reported that as (a/W) increases up to 0.8, results by the J integral method were shown less accurate than those by the direct displacement regression method.

The primary concern of this paper aims at illustrating the performance of the path independent J integral method when combined with the BEM under coarse mesh arrangements as an alternative to the finely meshed FEM. A boundary element scheme using the traction singular quarter point element is employed to obtain necessary accurate solutions along the boundary including the crack surface. While the direct strategy to calculate the SIF by Martinez and Dominguez [7] utilizes only the local data within the crack tip elements from the BEM solutions, the current J integral method needs not only all the boundary data but also additional quantities derived from them.

The standard J integral representation by Rice[2] is adopted for the computation of the SIF in cracked sheets. A fundamental conservation law in elasticity which proves essential for computing the J integral is derived by considering the gradient of the strain energy function. The origin of the formulation related to the path independent integrals can be referred to the works by Knowles and Sternberg[3]. Subsequently, a representative J integral for a center crack problem of finite width in plane stress is derived for the numerical implementation using quadratic shape functions. J integrals for the single and the double edge crack problems are also derived and numerically calculated for the comparison purpose with the analytical solutions available in the literature.[10] Since approaches under coarse mesh arrangements may be strongly dependent on the size effect of crack tip elements, as suggested by Jia et. al.[9], it may be desirable to find the computed values of K_I which change very little as the ratio (L/a) varies.

The potential application of the current method is not limited to previous opening mode I, symmetric crack problems of isotropic materials with traction free crack surface, and may include handling two-dimensional problems subjected to crack surface loading.

2. Numerical Formulation of The J_1 integral

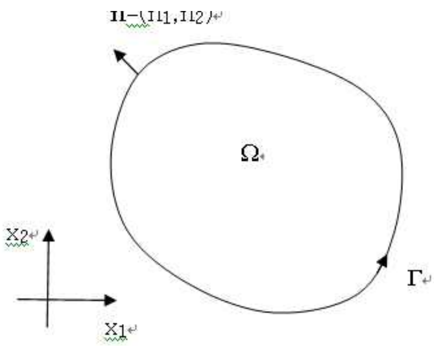

Figure 1. Domain Ω enclosed by a contour Γ of an elastic body.

Referring to Fig. 1, the well known J_1 integral is given as

$$J_1 = \oint_{\Gamma} (Wn_1 - \sigma_{ij} n_j u_{i,1}) d\Gamma \qquad (1)$$

When the near tip behaviors of the stress and the displacement are available in asymptotic form by Williams[1], they are substituted into the integrand of (1) and are integrated in the counterclockwise direction from the angle $-\pi$ to $+\pi$ for a circle of radius ε enclosing the crack tip. Resulting J_1 integral subjected to a limiting process $\varepsilon \to 0$ can be evaluated in terms of the stress intensity factor K_I and K_{II} for the cracked sheets.

$$J_1 = \frac{K_I^2 + K_{II}^2}{E} \quad \text{for plane stress} \qquad (2)$$

For the mode I opening fracture mode, the similar operation can be made for a contour enclosing the remote boundary of the cracked sheet. If this operation can be numerically performed utilizing the boundary data obtained from numerical analyses such as BEM or FEM, the stress intensity factor K_I can be immediately calculated from it.

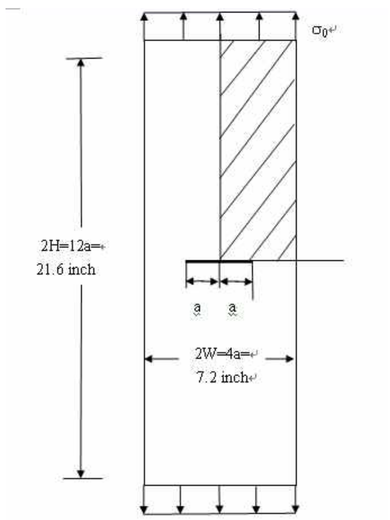

Figure 2. Central crack problem.

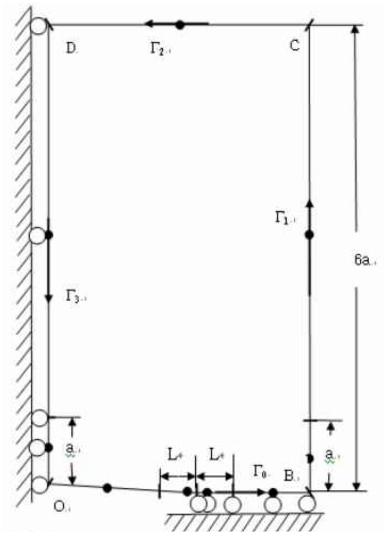

Figure 3. A 9-element mesh for a central crack problem.

A center crack problem in plane stress as considered by Blandford et al.[6] is analyzed again to illustrate the usage of the path independent J_1 integral. From the symmetry of the problem, it is sufficient for only one-fourth of the sheet to be discretized as shown in Fig 2. The problem of the double edge cracks can be treated in the similar way.

The contour in the counter clockwise direction consists of four pieces named with subscripts as shown in Fig 3. The corresponding J_1 integral is taken as

$$J_1 = 2 \oint_{\Gamma_1+\Gamma_2+\Gamma_3} \left[W n_1 - (\sigma_{11} u_{1,1} + \sigma_{21} u_{2,1}) n_1 - (\sigma_{12} u_{1,1} + \sigma_{22} u_{2,1}) n_2 \right] d\Gamma \ . \qquad (3)$$

It is noted that on the contour including the crack surface Γ_0 , where n_1=0, σ_{12}=0 and $u_{2,1}$=0, the integrand identically vanishes. The boundary conditions and the geometric conditions to evaluate the J_1 integral along the contour are summarized as

$$
\begin{aligned}
&n_1 = +1 ,\ n_2 = 0 ,\ \sigma_{11} = 0 ,\ \sigma_{12} = 0 \quad \text{on } \Gamma_1 \\
&n_1 = 0 ,\ n_2 = +1,\ \sigma_{22} = \sigma_0 ,\ \sigma_{12} = 0 \quad \text{on } \Gamma_2 \\
&n_1 = -1 ,\ n_2 = 0 ,\ u_1 = 0 ,\ \sigma_{12} = 0 \text{ on } \Gamma_3 .
\end{aligned}
\tag{4}
$$

Then, using the values of the unit normal vectors in (4), the J_1 integral is simply expressed as follows

$$
\begin{aligned}
\frac{J_1}{2} &= \int_{\Gamma_1} \frac{1}{2}\sigma_{22}\varepsilon_{22} d\Gamma - \oint_{\Gamma_2} \sigma_{22} u_{2,1} d\Gamma + \oint_{\Gamma_3} \left(-\frac{1}{2}\sigma_{11}\varepsilon_{11} - \frac{1}{2}\sigma_{22}\varepsilon_{22} + \sigma_{11} u_{1,1} \right) d\Gamma \\
&= \int_{\Gamma_1} \frac{1}{2}\sigma_{22}\varepsilon_{22} d\Gamma - \oint_{\Gamma_2} \sigma_{22} u_{2,1} d\Gamma + \oint_{\Gamma_3} \left(\frac{1}{2}\sigma_{11}\varepsilon_{11} - \frac{1}{2}\sigma_{22}\varepsilon_{22} \right) d\Gamma .
\end{aligned}
\tag{5}
$$

It is necessary for the above integration to be transformed into a suitable form for the numerical computation of the J_1 integral. To this end, isoparametric quadratic shape functions are employed as

$$
\phi_1(\eta) = \frac{1}{2}\eta(\eta - 1) ,\ \phi_2(\eta) = 1 - \eta^2 ,\ \phi_3(\eta) = \frac{1}{2}\eta(\eta + 1) \text{ for } -1 \le \eta \le +1 . \tag{6}
$$

It is recommended to use the same shape functions used in the numerical analysis. The interpolated coordinates, displacements and tractions in boundary elements are expressed as

$$
x_i^{(e)} = \sum \phi_k(\eta) x_i^{k,(e)} ,\ u_i^{(e)} = \sum \phi_k(\eta) u_i^{k,(e)} ,\ t_i^{(e)} = \sum \phi_k(\eta) t_i^{k,(e)} \tag{7}
$$

where i and (e) denote the node number and element number, respectively. The Jacobian along the contour is defined by

$$
J(\eta) \equiv \frac{d\Gamma}{d\eta} = \sqrt{\left(\frac{dx_1}{d\eta}\right)^2 + \left(\frac{dx_2}{d\eta}\right)^2} \tag{8}
$$

The differentiation of the displacement with respect to the coordinate x_j is given as

$$u_{i,j}^{(e)} = \frac{d\,u_i^{(e)}}{d\eta}\frac{d\eta}{dx_j} \tag{9}$$

Resulting J_1 integral is given as

$$\begin{aligned}\frac{J_1}{2} &= \frac{1}{2}E\sum_{\substack{\text{element}\\ \text{on }\Gamma_1}}\int_{-1}^{+1}\left(\frac{du_2^{(e),k}}{d\eta}\right)^2 J^{-1}d\eta + \sigma_0\sum_{\substack{\text{element}\\ \text{on }\Gamma_2}}\int_{-1}^{+1}\frac{du_2^{(e),k}}{d\eta}d\eta \\ &+ \frac{(1-\nu^2)}{2E}\sum_{\substack{\text{element}\\ \text{on }\Gamma_3}}\int_{-1}^{+1}t_1^{(e),k}t_1^{(e),k}Jd\eta - \frac{1}{2}E\sum_{\substack{\text{element}\\ \text{on }\Gamma_1}}\int_{-1}^{+1}\left(\frac{du_2^{(e),k}}{d\eta}\right)^2 J^{-1}d\eta \\ &- \nu\sum_{\substack{\text{element}\\ \text{on }\Gamma_3}}\int_{-1}^{+1}t_1^{(e),k}\left(\frac{du_2^{(e),k}}{d\eta}\right)d\eta\end{aligned} \tag{10}$$

This final expression containing numerical integrations of polynomial terms of was obtained and can be numerically evaluated using the Gaussian quadrature. It is noted that the path independent J_1 integrals for the problems of the double edge crack as shown in Fig. 4 and the single edge crack as shown in Fig. 5 in plane stress can be similarly obtained.

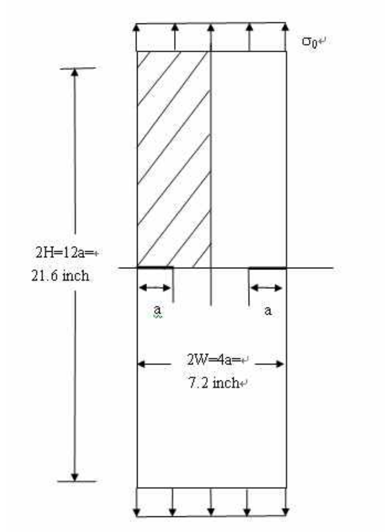

Figure 4. Double edge cracks.

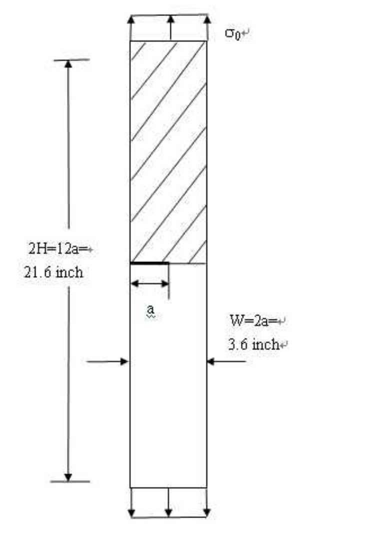

Figure 5. Single edge crack.

3. Traction Singular Quarter Point Elements

Referring to Fig. 6, crack tip elements are constructed using a quarter point element and a traction singular quarter point element. In the BEM formulation isoparametric with respect to the coordinate and the displacement, a mid point in both side of elements around the crack tip should be placed at a quarter of the element length on the straight boundary as shown in Fig.5. By doing so, $r^{+1/2}$ behavior of the displacement near the crack tip can be introduced. However, since , in the BEM, displacements and tractions are represented independently, traction shape functions possessing $r^{-1/2}$ behavior of the stress near the crack tip can be obtained by dividing each quadratic shape functions by $(r/L)^{+1/2}$ as explained by Martinez and Dominguez.[7]

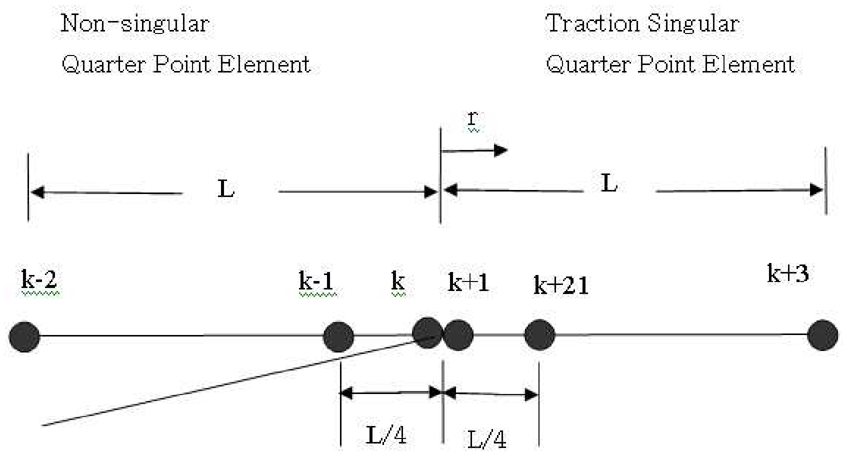

Figure 6. Crack tip modelling.

For this purpose, if a mid point in the right hand side quarter point element is placed at a quarter of the element length on the straight boundary, a geometrical relation between the coordinate and the variable r holds as follows

$$r = \phi_1(\eta) \bullet 0 + \phi_2(\eta) \bullet \frac{L}{4} + \phi_3(\eta) \bullet L = \frac{L}{4}(\eta + 1)^2 \tag{11}$$

Where quadratic shape functions defined in (6) were used. Its Jacobian is given as

$$J = \frac{dr}{d\eta} = \frac{L}{2}(\eta + 1) \tag{12}$$

The denominator to construct a traction singular quarter point element is established as

$$\sqrt{\frac{r}{L}} = \frac{1}{2}(\eta + 1) \tag{13}$$

By dividing the quadratic shape functions (6) with this, we have

$$\bar{\phi}_1(\eta) = \frac{\eta(\eta - 1)}{(\eta + 1)}\ ,\ \bar{\phi}_2(\eta) = 2(1 - \eta)\ ,\ \bar{\phi}_3(\eta) = \eta \text{ for } -1 \le \eta \le +1\ . \tag{14}$$

It is obvious that if the coordinate approaches -1, i.e., r->0, $\bar{\phi}_1(\eta)$ is singular with the order of the magnitude O($r^{-1/2}$).
Corresponding tractions are described as

$$t_i^{(e)} = \sum \bar{\phi}_k(\eta) t_i^{k,(e)} \tag{15}$$

As η approaches -1, 0, +1, respectively, the singular traction at the crack tip in the x_2 direction becomes

$$t_2^{(e)} \to \lim_{\eta -> -1} \frac{2}{(\eta + 1)} t_2^{1,(e)} = \lim_{r -> 0} \sqrt{\frac{L}{r}}\, t_2^{1,(e)} \to \frac{K_I}{\sqrt{2\pi r}}$$

$$t_2^{(e)} = 2\, t_2^{2,(e)} \text{ as } \eta \to 0,$$

$$t_2^{(e)} = t_2^{3,(e)} \text{ as } \eta \to +1. \qquad (16)$$

Thus, the stress intensity factor for the traction singular quarter point element can be directly obtained as

$$K_I = \sqrt{2\pi L} t_2^{1,(e)} \qquad (17)$$

This method has been first devised by Martinez and Dominguez.[7] As indicated by them, this direct method makes use of the nodal value of the traction at the tip, allowing unbounded behavior of the stress near the crack tip. Additionally, it is noted that while the l.h.s limiting value at the crack tip vanishes due to the traction free boundary condition, the r.h.s limiting value is unbounded with the order of the magnitude $O(r^{-1/2})$. Thus, it should be necessary to model the crack tip using the double nodes which makes enable to allow the continuity of the displacements and the discontinutity of tractions at the crack tip as shown in Fig.6.

4. Numerical Examples and Discussion

Three problems having individually different ratios of H/W and a/W were investigated using the approach described in this paper. The first two problems were taken from those by Blandford et al. [6] (H/W=3.0 and a/W=0.5 with a=1.8 in). The same problems were also investigated by Martinez and Dominguez.[7], and Jia et al[9]. The third problem was taken from that by Eischen[8] where the parameters are given as H=2.0 in, W=1.0 in, and a=(0.0,0.2,0.3,0.4,0.5,0.6,0.7,0.8) in. Unlike previous studies, the parameters used in this study are taken with H/W=2.0, a/W=0.5, 0.7, 0.8 for the center crack and the single edge crack problem and H/W=2.0, a/W=0.5, 0.8 for the double edge crack problem. The dimensions of each model under consideration for numerical computations are given as the width W and the height H as shown in Fig. 2, Fig. 4 and Fig. 5, respectively. The traction σ_0 at the remote boundary is taken unity. The material properties are taken with Young's modulus E=10,000ksi and Poission ratio =0.3.

Discretized boundary elements for a sheet with a center crack are illustrated in Fig. 3, where L is the length of each of the two adjacent elements including the crack tip. The sizes of elements adjoining with the corner in the l.h.s and the r.h.s of the rectangular sheets are taken with H/10 which is different from H/6 in prior studies [6,7,9]. For the problems of a/W=0.5, the boundary is discretized in a symmetric pattern with respect to the crack tip. For a/W=0.7 or 0.8, the quarter point element on the traction free crack surface against the traction singular quarter point element is configured to have the same length of element, i.e., L. The remaining part adjacent to the crack tip elements is filled with a single ordinary mid point quadratic element. Computed SIF's are plotted in terms of relative errors defined as $\Delta\%=\{K_I\ (\text{BEM})-K_I\ (\text{analyt.})\}/K_I$ (analyt.) in per cent. Finally, as suggested by Jia et. al.[9], it may be desirable for the computed values of K_I to change very little as the ratio (L/a) varies. To this end, all the numerical tests using 9 elements arrangement were conducted to examine the sensitivity of K_II for the range $0.05 \leqq (L/a) \leqq 0.9$. Relative comparisons are made between the two methods of the indirect J integral method and the direct one. Although both methods share the same BEM solution to compute the SIF, the results may be inherently different each other.

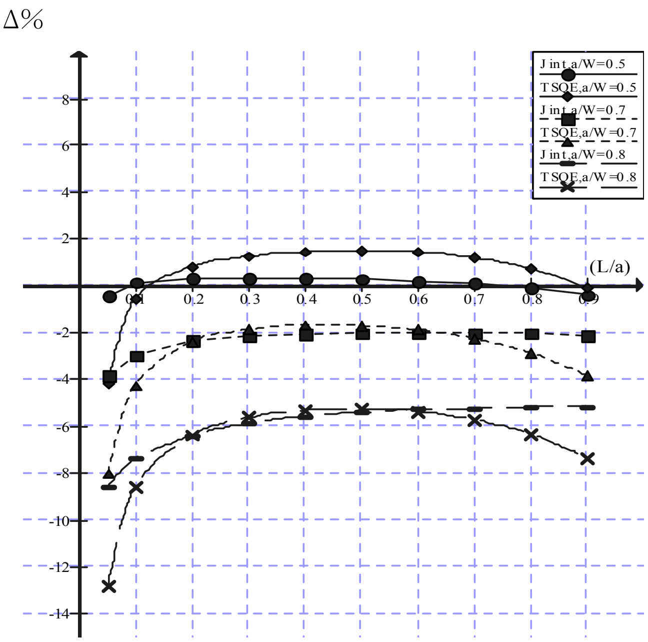

Figure 7. Center crack.

Center Crack

The lengths of center crack shown in Fig. 2 are taken with a=0.5, 0.7 and 0.8 in, respectively. For a/W=0.5, 0.7 and 0.8, the mode I SIF′s, referring to the Handbook by Tada[10], are given as K_I=1.4873, 2.2069 and 2.8790, respectively. The J integral method for a/W=0.5 in Fig. 5 presents quiteaccurate results within 0.5% of the exact value for the range 0.05≦ (L/a) ≦0.9 and is less mesh-sensitive than those of the direct method using the traction singular quarter point element. It is noted that although the pattern of K_I for a/W=0.5 when used with the traction singular quarter point element is not similar to that by Martinez and Dominguez.[7], the range of the error is bounded with sufficient accuracy. For (a/W)=0.7, both methods also show overall good accuracy within 2.5% of the exact value for the range 0.2≦ (L/a) ≦0.7. Both results for (a/W)=0.8, however, deviate at least 5.0% off from the exact value given by Tada.[10] The range in-sensitive to (L/a) for the direct method tends to be narrowed compared with the indirect one as (a/W) increases. For small (L/a), both methods show commonly degraded. To avoid such deterioration of K_I, Jia et al used an extra mid point quadratic element at each side of the crack tip element to make an 11-elemet mesh for (a/W)=0.5, obtaining errors under 1.5% for the center crack problem. In contrast, as can be seen in Fig. 7, the current J integral method for (a/W)=0.5 provides more excellent accuracy of error under 0.5 % without addition of extra elements. Results for (a/W)=0.7 and 0.8, show considerably improved accuracy compared with the direct method for small (L/a). Using the J integral method, it is obviously seen that the deterioration of K_I as (L/a) approaches 0.9 disappears.

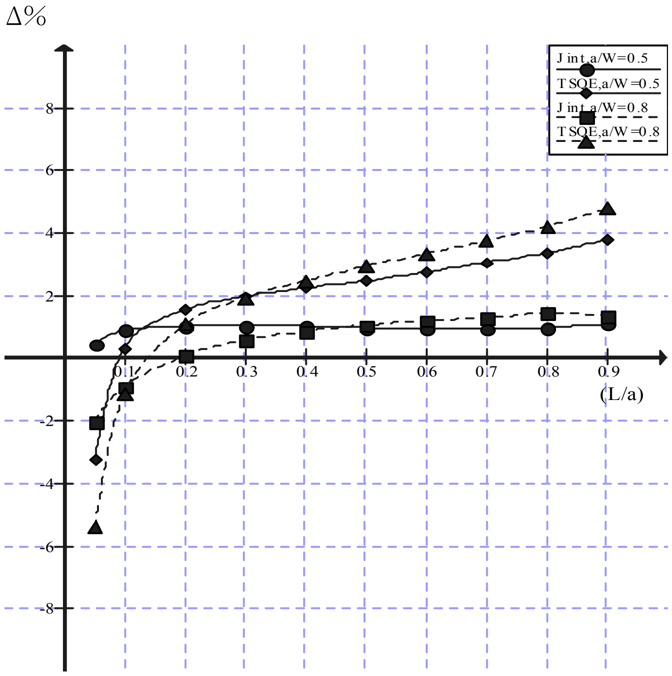

Figure 8. Double edge cracks.

Double Edge Crack

A double edge crack problem of its width, 2W=2.0 in and height 2H=4.0 in as shown in Fig. 4 is examined to calculate the stress intensity factor K_I. The lengths of crack are given with a=0.5 and 0.8 in, respectively. For a/W=0.5 and a/W=0.8, the mode I stress intensity factors are given as K_I=1.4690 and K_I=2.4044, respectively. It is noted that although the pattern of K_I for a/W=0.5 when used with the traction singular quarter point element is not similar to that by Martinez and Dominguez.[7], the range of the error is bounded with sufficient accuracy. The J integral method as in Fig. 8 for a/W=0.5 and a/W=0.8 shows quite accurate results within 1.1% and 2.1% over all the range of the parameter (L/a) between 0.05 and 0.9, respectively. Jia et al's attempt using 11-element scheme to avoid the degradation of K_I for small (L/a) =0.2 yields the errors under 1%. The current J integral approach also provides similar accuracy of error under 1 % without addition of extra elements. It is obvious that the J integral method shows considerably improved accuracy for a small (L/a) ratio. In particular, since the direct method yields increasingly inaccurate for (L/a)=0.1, it may be difficult to find the value of K_I to change very little as (L/a) varies.

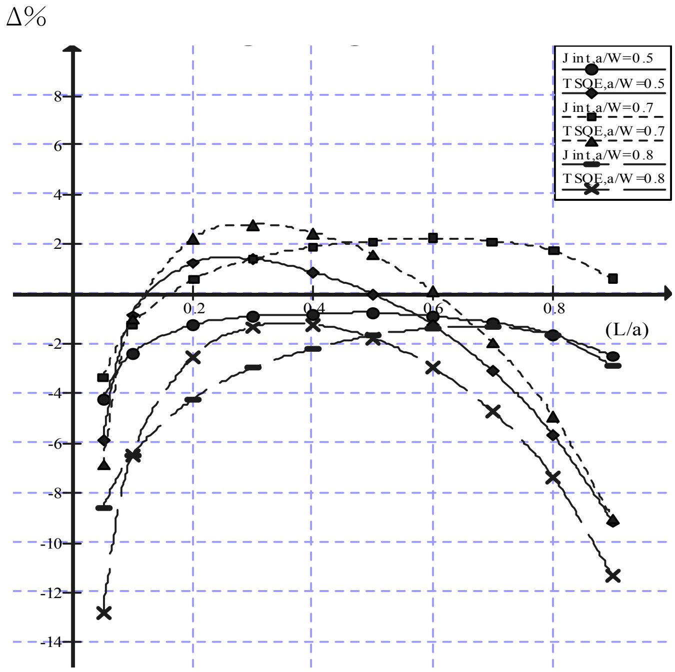

Figure 9. Single edge crack.

Single Edge Crack

The SIF K_I is calculated for the single edge crack problem of its width, W=1 in and height 2H=4 in as shown in Fig. 3. The lengths of crack are taken with a=0.5, 0.7 and 0.8 in, respectively. For a/W=0.5, 0.7 and 0.8, the mode I SIF's are given as K_I=3.55, 9.42 and 18.94, respectively.

The J integral method as in Fig. 9 for a/W=0.5 revealed relatively more accurate and flatter results around 1.0% over the range 0.3= (L/a) =0.6 than those of the traction singular quarterpoint element. For a/W=0.7, both methods show errors within 3.0% for the range 0.2=(L/a)=0.7. For a/W=0.8, while the J integral method yields 3% error over the range 0.3= (L/a) =0.9, the direct method shows 3% error over the range 0.2= (L/a) =0.6. Both methods show that as (L/a) varies the sensitivity of K_I vanishes on certain interval of (L/a). In particular, the J integral method yields flatter curve over the wide band of (L/a), and appears to be less influenced due to the size effect of the crack tip elements than the direct one.

5. Concluding Remarks

The integral form suitable to computing two-dimensional stress intensity factors in the linear elastic fracture mechanics has been derived from a conservation law of the elasticity, leading to the standard J integral. Problem dependent numerical formulations to calculate the J_1 integral have been made using the BEM solutions by employing quadratic shape functions. It was known that on the contour including the crack surface 0, the integrand of the J_1 integral identically vanishes regardless of the mesh arrangement around the crack tip. The strain term ε_{11} appearing on the contour Γ_3 generated from the symmetry axis of the given problem is considered as an internal quantity which cannot be directly obtained from the BEM solution. In subsequent numerical derivation utilizing linear elastic Hooke's law under the plane stress to condition, it was shown that it can be expressed in terms of the boundary traction $t_1()$ and the derivative of the boundary displacement $u_2()$ with respect to x_2.

The J integral method combined with the traction singular quarter point element for the three crack problems was examined for comparisons with the direct method using the traction singular quarter point element. It turned out that the indirect J integral method produced uniformly accurate results over the wide band of the parameter (L/a) compared with the direct one based on the traction singular quarter point element. While the direct method showed serious deterioration of K_I for the extreme values of (L/a)<0.1, the J integral method yielded considerably improved results. Finally, it is concluded that though under coarse mesh arrangements the J integral method shows relatively little dependence on the size effect of the crack tip elements than the direct one, the sensitivity analysis should necessarily be conducted for the accurate determination of stress intensity factors.

Reference

[1] Williams ML, On the stress distribution at the base of a stationary crack, ASME Journal of Applied Mechanics 1957;24:109-114.

[2] Rice JR. A Path-Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. ASME Journal of Applied Mechanics 1968;35:379-386.

[3] Knowles JK, Sternberg E. On a Class of Conservation Laws in Linearized and Finite Elastostatics. Archives for Rational Mechanics Analysis 1972;44:187-212.

[4] Henshell RD, Shaw KG. Crack Tip Finite Element Are Unnecessary. International Journal for Numerical Methods in Engineering 1975;9:495-507.

[5] Barsoum RS. On the Use of Isoparametric Finite Elements in Liniea Fracture Mechanics. International Journal for Numerical Methods in Engineering 1976;10:25-37.

[6] Blandford GE, Ingraffea R, Liggett JA. Two-dimensional Stress Intensity Factor Computations using the Boundary Element Method. International Journal for Numerical Methods in Engineering 1981;17:387-404.

[7] Martinez J, Dominguez J. Short Communication:On the Use of Quarter-Point Boundary Elements for Stress Intensity Factor Computations. International Journal for Numerical Methods in Engineering1984;20:1941-1950.

[8] Eischen JW. Fracture of Nonhomogeneous Materials. Ph.D. Mechanical Engineering Thesis. Stanford University Jan. 1986.

[9] Jia ZH, Shippy DJ, Rizzo FJ. On the Computation of Two-Dimensional Stress Intensity Factors Using the Boundary Element Method. International Journal for Numerical Methods in Engineering 1988;26:2739-2753.

[10] Tada H, Paris PC, Irwin GR. The Stress Analysis of Cracks Handbook. Del Research Corporation Hellertown, Pennsylvania, 19l.